The probability that a pair p_a of people from a set ot n people have the same birthday is the same as the probability p_b of another pair of people who have the same birthday:

$$p_a = p_b = \frac{n! \cdot \binom{365}{n}}{365^n}$$

The probability that the two pairs a and b have a *common* birthday is $p_c = p_a \cdot p_b$:

$$\begin{aligned}
p_c &= p_a \cdot p_b \\
&= \left(\frac{n! \cdot \binom{365}{n}}{365^n} \right)^2 \\
&= \frac{n! \cdot \binom{365}{n}}{365^n} \cdot \frac{n! \cdot \binom{365}{n}}{365^n} \\
&= n! \cdot \frac{\frac{365!}{n! \cdot (365-n)!}}{365^n} \cdot n! \cdot \frac{\frac{365!}{n! \cdot (365-n)!}}{365^n} \\
&= n! \cdot \frac{365!}{n! \cdot (365-n)! \cdot 365^n} \cdot n! \cdot \frac{365!}{n! \cdot (365-n)! \cdot 365^n} \\
&= \frac{365!}{(365-n)! \cdot 365^n} \cdot \frac{365!}{(365-n)! \cdot 365^n} \\
&= \frac{365!^2}{(365-n)!^2 \cdot 365^{2n}}
\end{aligned}$$